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Simulating the Dynamics of Urban Impact on Middle America from 1600 to the Year 2000

Ten years ago we presented a paper in the research methodology session of the first CLAG conference in which we described two aspects of quantitative methodology: uniform areal data units and analog simulation. Now we want to follow up on one of those two aspects, analog simulation. More correctly, we would like to follow up on only the second half of that aspect, simulation, since most of the work we are now doing is no longer on an analog system but on a digital system.

In 1970 we described two simulation systems: analog field plotters and resistance networks. Analog field plotters involve drawing a map of the study area on paper that is electronically conductive. Electric current is induced at the location of cities in amounts proportional to the sizes of the cities. Electric potential is plotted on the map with a single probe. The electric potential is directly analogous to Warntz-type population potential maps. The field plotter also demonstrates the theoretical hexagonal trade regions of Christaller, produces other shapes of trade regions depending on the size and spacing of towns, and solves problems of stream profiles, great circle routes, atmospheric pressure, water pressure in aquifers, the impact of highways, and accessibility measures under different locational constraints. In general, the field plotter permits the solution of the complex LaPlace and Poisson equations that underlie most concepts of spatial interaction. Breakthroughs in electronic instrumentation and modeling materials made the field plotter suitable for operation by persons who have no background in either electronics or mathematics.

The resistance network is a more complex analog simulation model. Such a network replaces the paper map of the field plotter with a matrix of nodes connected by electric wires. The continuous distribution of the paper is given up for the increased control of the resistance network. Roads and barriers may be erected with virtually complete control. The resistance network also permits many more inputs than the field plotter. Perhaps the major advantage is that the

resistance network can be programmed to solve non-steady-state problems; the field plotter can only approximate such dynamic problems by a sequence of models. But the resistance network is difficult to use because it takes about 50 hours to program a problem and 20 hours to read out the results, it requires a knowledge of electronics to operate, and it is not available commercially.

We argued in 1970 that the field plotter could make it possible for a geographer, or an average freshman student, to apply a rigid methodology and sophisticated theory without knowledge of either mathematics or electronics. In other words, it can bridge the gap between the so-called traditional geography and the quantifiers. That is still our goal. In 1972 we received from the National Science Foundation the funds to develop a system which could be easily operated like the field plotter and capable of non-steady-state simulations of gravity-like models like the resistance networks. We developed the MAPS (Multi-dimensional Analysis and Processing Spaces) system, the first successful raster graphics system to refresh out of random access memory. In other words, MAPS used a digital computer to keep track of the color that each resolution cell (pixel) on a color television tube would be each 30th of a second that the tube is scanned. It succeeded magnificently but it was very expensive and available only in our laboratory.

In 1977 we closed down the MAPS laboratory to concentrate attention on CHILD Incorporated, a private research and development laboratory, in order to design such equipment in a way that it could be made available for manufacture and marketing and, thus, available to the profession. During 1979 we spent the entire year helping ADI, an Ann Arbor, Michigan firm, get into production the LIGHT display generator we designed; it is now available. Several other companies are developing similar devices. SIGGRAPH, ACM's Special Interest Group on Computer Graphics and Interactive Techniques (Association for Computing Machinery SIGGRAPH, 1133 Avenue of the Americas, New York, NY 10036) is the industry organization for computer graphics and they can provide a current list of products and manufacturers.

Thus, in 1980 we are developing our first commercially available system to simulate the dynamic impact of urban areas over time. In late 1979 we got the program running on simple test data. Now, in early 1980, we are setting up our first problem and it is the simulation of urban impact on Middle America 1600 to 2000. The first public showing of our preliminary efforts was scheduled for the

1980 CLAG session. We spent most of our efforts on getting the model to run and, consequently, were less professional than we would have liked in gathering the data for model. Students, mostly undergraduate, in our Latin American geography class gathered the data as a class project. Your suggestions for making the data base more accurate would be most appreciated.

The system is designed to illustrate the spread of urban impact of cities of 10,000 or more population for each decade since 1600, with the data points being updated each 50 years. To understand how the urban impact is measured you will need. to understand two processes; heat flow on a metal map and four-way averaging on a set of Spanish tiles.

Heat Flow on a Metal Map

We ask that you imagine three concepts. First, imagine that cities are sources of energy that spread out from each source just as heat spreads out from a spot on a sheet of metal when the spot is being heated. Second, imagine that we have a large metal map of Mexico; Central America; northern South America; and, suspended in the area cut out where the Caribbean Sea occurs, the major islands. The map measures 256 feet east and west and 256 feet north and south, and has an electric heater under the site of each city presently over 10,000 in size, and a thermostat that allows us to control the heat under each site. Third, imagine that we have a thermometer on each square foot of the metal map, and that we can record the temperatures to the nearest tenth of a degree for all 65,536 thermometers at the end of each minute of lapsed time.

With such a setup in our imaginary laboratory, we can begin the simulation. The initial temperature on each square foot of land is 25 degrees Celsius. We raise the temperature of Mexico City one degree because it was the only city on our map we are treating as having a population of ten thousand in 1600. The heat energy of Mexico City begins to spread in all directions and, at the end of each minute, we record the temperature of each square foot of the metal map. We assume that the passing of each minute of real time represents the passing of a year of simulated time. Thus, after fifty minutes we call the temperature readings the population potential for 1650. It is this population potential, we argue, that represents the theoretical impact of urban areas on the rural landscape.

We need to explain the concept of a dynamic equilibrium. The metal map had a temperature of 25 degrees Celsius everywhere; it was dissipating heat energy at

the same low rate that it was receiving it. But we disturbed the map's energy system by applying additional heat at one site. No matter how much energy is dissipated, the temperature at that site does not fall below the temperature we set. Heat spreads in all directions from Mexico City, causing nearby areas to have higher temperatures and, consequently, causing them to dissipate more heat energy into the surrounding air. Given enough time, the heat spreads until, once again, the amount of heat energy dissipated from the entire map equals the amount being received, but the temperatures across the map are no longer a uniform 25 degrees. When no further change in the pattern of potential is experienced with the passage of time, a dynamic equilibrium has been achieved.

Before a dynamic equilibrium is achieved, we change the inputs to conform to the sizes of cities for 1650. We turn the heaters up one degree for each 10,000 people; consequently, a city of 100,000 would be ten degrees hotter than normal. The simulation continues with the inputs for each successive half century reaching dynamic equilibrium. By the nineteenth and twentieth centuries the amount of momentum in the model increases enormously with each successive half-century. By letting the simulation run for 30 seconds past the 1950 update we can simulate the present situation. By letting it run beyond 1980 we can simulate the likely future developments. The momentum of the system has an impact that reaches well into the future.

The final result of the simulation is a metal map that is red hot in some places but still at 25 degrees Celsius in other places. More important is that we have a set of maps that will let us trace the changing temperature over time. Recent applications of gravity model concepts still work exclusively under dynamic equilibrium (steady state) assumptions. We feel that in the dynamic world of modern human settlement, population tends to respond to the gravity model processes, but external forces continually change some of the inputs and initiate new transient stages. There is little literature to guide us; we rely heavily upon intuition. We are just beginning to develop the computational facility to handle such simulations. The procedure employed is not identical to that of the heat diffusion model just described, but it is close to it. We use a system that involves averaging the values of neighboring areas. To illustrate the procedure we need a different map.

Averaging on Tile Maps

Imagine another map of Middle America at the same scale, this one being a ceramic map made up of 256 rows and 256 columns of tiles, each tile being one square foot in size. In the center of each tile is a number which records its population potential. Underneath that number is space for a horizontal line; the line is drawn on the tile when the average potential of the four adjacent tiles is one or more units larger than that number.

On such a map we can rerun the simulation. There are two steps in each iteration of the model. First, we update the potentials for each of the tiles from the previous iteration. Second, we compute the average potentials for adjacent tiles, we draw a line on each tile whose value is exceeded by the average value of adjacent tiles, and we remember the average value. It is that average value that will be used as the new value for that cell when we update for the next iteration.

We arbitrarily designate two iterations to represent one year. We start in 1600 by assigning the tile containing Mexico City a value of 10,000. As before, the potential of that tile will not be permitted to drop below its input value. After 100 iterations we call the resulting pattern of values the population potential for 1650. We then increase the size of Mexico City to 20,000 and add Antigua Guatemala Bogota, Havana, and Santo Domingo at 10,000 to represent our estimate of their 1650 urban populations. After another 100 iterations we, again, increase the input values to represent our estimates of the 1700 urban populations and we add five new urban areas: Puebla, Quito, Caracas, Matanzas, and Santiago de Cuba.

We proceed in that way, updating each 50 years, until the 1950 values have run for 60 iterations and we declare that to be the present population potential. Another 40 iterations carry the simulation to its conclusion at the year 2000, although the model still has enormous momentum at that time, left from the 1950 and earlier inputs. The results closely approximate the results that would be obtained by running the metal map experiment, and they can be obtained much more easily.

A Sample Problem

To do the arithmetic necessary to set up and compute accurately the 800 iterations required to run the 256 by 256 simulation without the aid of a computer would require more time and effort than we would be willing to spend. But to understand such a model, one needs to work by hand at least a sample

problem. At least, one should have a worked-out simplification on which the computational procedure could be verified.

At the end of the present report are four pages of iterations on a 9 x 9 matrix where the only input is in the centermost cell. the input has a value of 100, represented by "XX".

Note that the initial condition indicates that, in the first iteration, XX will spread to four cells, indicated by the four horizontal lines. Their value, as indicated on the first iteration, is (100 & 0 & 0 & 0) divided by 4 - 25. The maximum change in value of a cell, a good index to follow in the model, is 25; we refer to it as the maximum delta and abbreviate it "MD." Note that eight new cells are affected in the second iteration and MD drops to 12. In iterations three, four, and five the number of cells in which values rise increases from 20 to 22, while MD drops from six to three. Note also that all the 20 cells that change in iteration three are unchanged in iteration four and vice versa; that is one reason for having a minimum of two iterations represent a unit of time -- it takes two iterations for each cell to have a chance to change. In iterations six through eleven the pattern becomes more organic and less geometrical in its growth. In iterations twelve through seventeen the pattern changes less and less. Fifty-two cells change value in twelve and thirteen while only twenty-four change in sixteen and seventeen. Iterations eighteen through twenty-one finish the series. No more changes occur and a dynamic equilibrium has been established. The equilibrium could be disturbed by assuming that iterations nineteen and twenty-one are two halves of one problem instead of two iterations in the same problem. If we assumed that the boundary between the two halves permitted nineteen to influence twenty-one but not vice versa then a new series, 02, would begin, run five iterations, and the changes in the bottom half are as displayed.

If we assume that the boundary between the two completely disappears, then yet another series, 03, begins, runs nine iterations and produces for the bottom half the last visual we present. Note that the interaction potentials in the last two outputs lose their perfect symmetry. In a complex problem most symmetry gets lost when potentials from the inputs begin to interact. If you understand this 9 x 9 sample problem then you will better understand the 256 x 256 model. If you understand the 256 x 256 model then you will better understand the geography of Middle America.

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